Harnessing critical incidents for learning

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A critical incident is a situation or event that holds significance for learning, both for the students and teachers. It is "unplanned, unanticipated and uncontrolled" (Woods, 2012, p.1). Successfully using critical incidents in a classroom situation provides opportunities for rich analysis of classroom practices. The purpose of this article is to discuss how critical incidents can be harnessed for students' and teachers' development.

Teaching is multifaceted; it is not linear, meaning it cannot be fully prepared beforehand. Even with the best of preparations, one cannot always anticipate whether a lesson will go as planned. As a result, a critical incident, namely an unpredicted incident that teachers had not anticipated beforehand, may occur. This is in line with Rowland's term of contingency; "...it concerns teachers' readiness to react to situations that are almost impossible to plan for." (Petrou & Goulding, 2011, p.18–19.)

This paper presents four examples of critical incidents from a Year 7 teacher's lesson excerpts in Indonesia involving teaching of fractions, to show how they shaped classroom situation, brought forward elements of conflict, and created learning opportunities. Three examples are drawn from the lesson using a web-based applet (Examples 1, 2 and 3). The illustration of these critical incidents will be followed by a discussion on how to harness them in order to develop students' understanding or be used as a challenge as well as a learning process for teachers.

The applet used in this lesson can be accessed through the following link: http://www.bbc.co.uk/skillswise/game/ma17frac-game-fractions-side-by-side. The applet enables the user to present fractions in symbolic and pictorial forms by dragging numbers onto the blackboards on each side of the screen (see Figure 1 and Figure 2). This applet has affordances and constraints. Some of the affordances are: the ability to present fractions in symbolic and pictorial forms at the same time; visualisation of fractions instantly in different models (namely pizza, people, a glass of water and a chocolate bar) and as a tool for explorations. The constraints are: this can only present proper fractions, namely fractions that are greater than or equal to zero but less than or equal to one, and it does

not allow zooming and overlaying. As a result of using this applet, comparing $\frac{8}{9}$ and $\frac{9}{10}$ becomes difficult, especially when comparing fractions whose numerators and denominators are larger numbers (e.g. $\frac{10}{11}$, $\frac{11}{12}$ and $\frac{200}{201}$).

The mathematics content associated with the following critical incidents align to Year 7 content descriptions in the *Australian Curriculum*, namely "Compare fractions using equivalence (ACMNA152)"; "Introduce the concept of variables as a way of representing numbers using letters (ACMNA175)"; and "Create algebraic expressions and evaluate them by substituting a given value for each variable (ACMNA176)".

Critical event 1

"Which one is greater?" "Which one is smaller?" These were the questions posed by the teacher when asking her students to compare three pairs of fractions: $\frac{1}{4}$ and $\frac{1}{10}$, $\frac{3}{8}$ and $\frac{7}{8}$, $\frac{2}{9}$ and $\frac{8}{7}$. Students chorused their answers, resulting in correct responses for the first two pairs of fractions, and incorrect responses for the third pair of fractions i.e., $\frac{2}{9} > \frac{8}{7}$. When the teacher directed her students to check their answers using a web-based applet, video data revealed that the students were able to verify their initial thought that $\frac{1}{4} > \frac{1}{10}$ and $\frac{7}{8} > \frac{3}{8}$ in less than a minute. However, when a group of students attempted to visualise the fraction $\frac{8}{7}$, a pop-up note appeared saying, "That fraction is more than 1. It is still a fraction, but we can't show it in this picture." Without even attempting to read it, the pop-up note was closed by a student by clicking the icon. An empty box was then displayed on the screen (see Figure 1), which was later pointed out by a member of the group, "I told you so, that's empty so two ninths is greater."

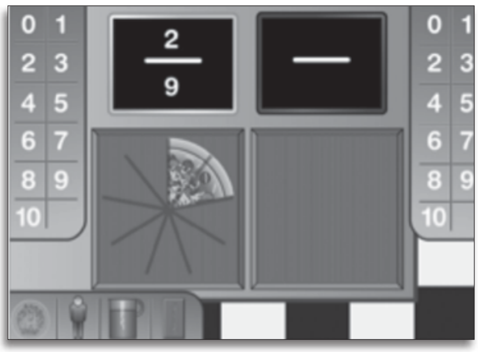

Figure 1

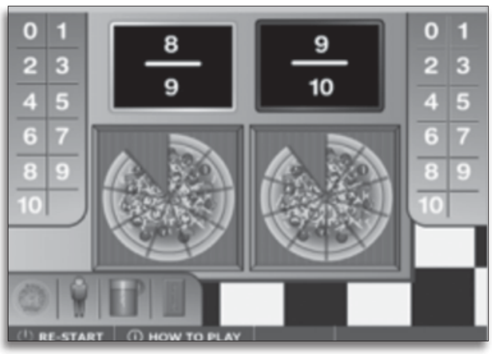

Figure 2

In the case discussed here, one may argue that the choice of task assigned by the teacher is not appropriate because $\frac{8}{7}$ cannot be represented by the applet. One may also criticise the design of this technology, such as the display of an empty box instead of an image of a large cross. However things like these, such as assigning a task whose consequences have not been fully anticipated, can possibly occur in any teaching situation. Similarly, the design of any technology may contain limitations, which become apparent when it is used.

The important questions are: how could teachers become cognizant and be able to identify and make use of such critical incidents? How to take advantage of the critical incidents to (1) reflect and re-examine the mathematical content or the pedagogical decisions, and (2) resolve in-the-moment students' difficulties in the classroom or to meet the emerging learning needs of students during the learning process?

The critical incident in which several students found that $\frac{2}{9} > \frac{8}{7}$ shows among other things that they do not understand the meaning of these symbols or the relationship between the symbolic and pictorial representations of a fraction. Therefore, teachers can open a discussion or formulate new tasks such as asking each group to observe 'sevenths' (e.g., $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$) using the available applets in each group since this applet represents those fractions accurately and instantly. By directing children to utilise the applet and observe the changes in the models as the numerators change, children will be able to visualise the model of $\frac{8}{7}$ (which consists of eight sevenths) even if it cannot be represented on the applet. In other words, children will be able to picture $\frac{8}{7}$ as a fraction that is represented by more than one pizza. Another thing a teacher could do is ask the students to draw $\frac{8}{7}$ in their notebooks with models of their choice, or ask the students to write five other fractions that, if represented by the applet, will generate an empty box. Through these activities, students are made aware of the limitations of the applet and, at the same time, these limitations can become an opportunity for teachers to develop students' abstract thinking.

Critical event 2

The teacher assigned each group to investigate the question, "Which is greater, $\frac{a}{b}$ or $\frac{b}{c}$, where a, b, and c are consecutive non-negative integers." This task was different to the usual tasks given in the school textbooks. This was given to encourage students to use higher order thinking, create opportunities for them to explore different pairs of fractions using the applet, and to develop mathematical conjectures.

Once the students understood the intent of the problem, they took turns in operating the applet to represent several pairs of fractions, such as: $\frac{1}{2}$ and $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{4}{5}$. From these examples, they concluded that $\frac{b}{c}$ is greater than $\frac{a}{b}$. Some students later became uncertain of their conclusion when they attempted to represent three other pairs of fractions: $\frac{6}{7}$ and $\frac{7}{8}$, $\frac{7}{8}$ and $\frac{8}{9}$, $\frac{8}{9}$ and $\frac{9}{10}$ (see Figure 2). A student clearly stated that $\frac{6}{7}$ was "the same as" $\frac{7}{8}$. Such a constraint most likely emerged when the student found it difficult to distinguish pictorial representations of the fractions $\frac{6}{7}$ and $\frac{7}{8}$ in the applet. The teacher further asked her students, "Then what if the number isn't from there, such as $\frac{9}{10}$ and $\frac{10}{11}$?"

The dialogue below demonstrates a discussion between the teacher and the students. (S: Students, Ss: Students chorusing, (.): Waiting time)

S: It's the same, Miss.
T: The same?
S: It's sometimes the same, Miss.
T: (walks while smiling) The same?
S: It's sometimes the same. (The class is noisy and "ten elevenths" was also heard).
T: Can you try it on there (refering to the applet)?
(Asking for the case $\frac{9}{10}$ and $\frac{10}{11}$)
Ss: No, you can not.
T: What about the numbers that are larger than that?
(Teacher points at the two fractions on the board from afar.)
S: It's the same.
T: Is it always the same? Is $\frac{b}{c}$ always greater? (.)
S: Sometimes.

Similar to the first critical incident, one may also criticise that the task of choice is not in accordance with the existing tools, for it brings students to formulate incorrect conclusions. For example, when the students compared $\frac{8}{9}$ with $\frac{9}{10}$, they concluded that the two fractions are equal. This is prompted by the similar looking pictorial representation of the fractions and the inability of the applet to allow overlaying or zooming.

Another challenge present in the critical event is the change of image size. These changes give the impression that two pizzas will increasingly look similar when the numbers used as the numerators and denominators are increased. This very likely brings students to conclude that $\frac{a}{b}$ and $\frac{b}{c}$ are occasionally equal. Moreover, this task could turn into a chaotic situation because of the many opinions that may arise. As a result, it becomes very tempting to claim that a task like this should very well be avoided.

However, when further observed, this critical incident could possibly become a learning moment for both the teacher and the students. The teacher, for example, may feel challenged to prove how students reached such conclusions. Although the teacher does not expect the students to contribute a sophisticated proof, such as proving their conclusions algebraically, the teacher is able to think of a way to utilise the applet in developing the students' mathematical reasoning.

The answer, "sometimes equal", indicates that students have yet to adequately understand the meaning of fractions or the relationship between the symbolic and pictorial representations of fractions. It seems that students were able to correctly conclude $\frac{3}{4}$ to be greater than $\frac{2}{3}$ because of the easily distinguishable difference between the size of the pizzas. When comparing $\frac{8}{9}$ and $\frac{9}{10}$, the different sizes of the pizzas are difficult to distinguish. Therefore, students can be assisted to compare the fractions $\frac{1}{9}$ and $\frac{1}{10}$. In other words, this critical incident could become the basis for investigating unit fractions; to investigate the fact that the larger the denominator of a unit fraction, the smaller the fraction will be. This understanding of unit fractions can help students to compare any variation of $\frac{a}{b}$ and $\frac{b}{c}$, such as $\frac{2000}{2001}$ with $\frac{2001}{2002}$ by only comparing $\frac{1}{2001}$ with $\frac{1}{2002}$. Another possible decision is to facilitate students in taking advantage of the equivalent fraction concept or the cross multiplication method in a meaningful way. These ideas could be enhanced through the use of the applet because the applet can also represent equivalent fractions. For example, $\frac{1}{2}$ and $\frac{2}{3}$ are represented the same way as $\frac{3}{6}$ and $\frac{4}{6}$.

This critical event has become a learning moment for mathematics learners, showing that not all fractions can be easily represented through pictures. With the experience of pictorial representations, provided through the applet, teachers will be able to assist learners in constructing a mental model, which allows them to think abstractly and formulate a correct conclusion. This is one of the main objectives of learning mathematics.

Critical event 3

Another critical incident occurred during the investigation of $\frac{a}{b}$ and $\frac{b}{c}$ where a, b, and c are consecutive positive integers. Some of the students had reached the correct conclusion (that is $\frac{b}{c} > \frac{a}{b}$), but unfortunately with an incorrect reasoning. They used the number of pizza slices as a measure to decide which of two fractions is greater. For example, in comparing

$\frac{3}{4}$ and $\frac{4}{5}$, they decided that $\frac{4}{5} > \frac{3}{4}$ because the number of slices in the $\frac{4}{5}$ pizza model is greater than in the $\frac{3}{4}$ pizza model. This reasoning was also applied when the group compared $\frac{8}{9}$ and $\frac{9}{10}$ (see Figure 2). The students' conclusion when based on the given task (a, b, and c are consecutive) is locally correct. However, the danger of using number of pieces is that it may lead to a possible misconception. This is similar with the case of students who claims that "times always makes bigger".

A student's unanticipated conclusion can be utilised to develop the student's mathematical reasoning. The teacher can write the student's reasoning on the board (make it publicly available for all the students) and provide an opportunity for other students to justify the accuracy of the conclusion of comparing two fractions. The student's conclusion above cannot be used to compare any two fractions. Students can be prompted to prove the incorrectness of such reasoning. One counter example is the comparison of $\frac{3}{4}$ and $\frac{7}{15}$, that is $\frac{3}{4}$ is greater than $\frac{7}{15}$ although $\frac{7}{15}$ has more slices (i.e., 7 out of 15 slices) than $\frac{3}{4}$ (i.e., 3 out of 4 slices).

Discussion

Mathematics education researchers stress the importance of thinking about the possibility of errors or difficulties faced by students during the learning process (e.g., Ball, Thames, & Phelps, 2008). However, depending on the teacher's knowledge and experience, the teacher may not anticipate a number of difficulties or errors; some consequences might not have even been thought of.

This challenge can become more complex when teachers integrate technology in teaching mathematics. With technology, students have more opportunities to interact with mathematics and, at the same time, the possibility of unanticipated thoughts or critical incidents tend to be more likely to occur. As a result, the use of technology can cause teaching to become more complex than to teach without it.

This paper highlights the effectiveness of a web-based applet for displaying pictorial representations in an interactive manner. However, due to the teacher's inexperience in using the applet, the teacher had to face unanticipated incidents. We argue that these critical incidents need to be considered positively by teachers; they need to be considered as a challenge by the teachers themselves in re-thinking the extent to which mathematics can be represented through the selected technology.

To be able to recognise and take advantage of the critical incidents, the teacher requires mathematics content knowledge, knowledge on teaching methods and processes, and knowledge of the technology used. The combination of mathematics content knowledge and pedagogical knowledge is commonly referred to as 'pedagogical content knowledge' (PCK) (Shulman, 1986) while the combination of knowledge in content, pedagogy, and technology is termed 'technological pedagogical content knowledge' (TPCK) (Mishra & Koehler, 2006). Teachers with strong PCK and TPCK are more capable of identifying and taking advantage of the critical events occurring in their classroom. Reciprocally, the willingness to reflect on critical incidents gives teachers opportunities to consolidate their PCK and TPCK.

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